## Primordial Magnetic Fields from Dark Energy

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Evidences indicate that the dark energy constitutes about two thirds of the critical density of the universe. If the dark energy is an evolving pseudo scalar field that couples to electromagnetism, a cosmic magnetic seed field can be produced via spinoidal instability during the formation of large-scale structures.

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Recent astrophysical and cosmological observations such as dynamical mass, Type Ia supernovae (SNe), gravitational lensing, and cosmic microwave background (CMB) anisotropies, concordantly prevail a spatially flat universe containing a mixture of matter and a dominant smooth component with effective negative pressure [1]. The simplest possibility for this component is a cosmological constant. A dynamical variation calls for the existence of dark energy whose equation of state approaches that of the cosmological constant at recent epochs. Many possibilities have been proposed to explain for the dark energy. Most of the dark energy models involve the dynamical evolution of classical scalar fields or quintessence (Q). For a Q-model, the dynamics is governed by the scalar field potential such that the vacuum energy becomes dynamically important only at recent epochs and gives rise to an effective cosmological constant today. So far, many different kinds of scalar field potentials have been proposed. They include pseudo Nambu-Goldstone boson (PNGB), inverse power law, exponential, tracking oscillating, and others [2]. Upcoming observations will measure the equation of state so as to discriminate between these models and distinguish them from the cosmological constant [3].

Since the scalar potential  $V(\phi)$  of the Q field is scarcely known, it is convenient to discuss the evolution of  $\phi$ through the equation of state,  $p_{\phi} = w_{\phi} \rho_{\phi}$ . Physically,  $1 \geq w_{\phi} \geq -1$ , where the latter equality holds for a pure vacuum state. Lately some progress has been made in constraining the behavior of Q fields from observational data. A combined large scale structure (LSS), SNe, and CMB analysis has set an upper limit on Q-models with a constant  $w_{\phi} < -0.7$  [4], and a more recent analysis of CMB observations gives  $w_{\phi} = -0.82^{+0.14}_{-0.11}$  [5]. Furthermore, the SNe data and measurements of the position of the acoustic peaks in the CMB anisotropy spectrum have been used to put a constraint on the present  $w_{\phi}^{0} \leq -0.96$  [6]. The apparent brightness of the farthest SN observed to date, SN1997ff at redshift  $z \sim 1.7$ , is consistent with that expected in the decelerating phase of the flat  $\Lambda$ CDM model with  $\Omega_{\Lambda} \simeq 0.7$  [7], inferring  $w_{\phi} = -1$  for z < 1.7. In addition, several attempts have been made to test different Q-models [8]. Nevertheless, it is primitive to differentiate between the variations, and the reconstruction of  $V(\phi)$  would require next-generation observations.

Although the physical state of the dark energy can be probed through its gravitational effects on the cosmological evolution, it is important in fundamental physics to understand whether the quintessence is a nearly massless, slowing rolling scalar field. It has been pointed out by Carroll that the existence of an approximate global symmetry would allow a coupling of the Q field,  $\phi$ , to the pseudoscalar  $F_{\mu\nu}\tilde{F}^{\mu\nu}$  of electromagnetism, which provides a potential observable in polarization studies of distant radio sources [9]. As long as an ultralight  $\phi$  field couples to photon where for a slow-roll condition, the mass  $m_{\phi}$  is comparable to  $H_0$ , it is conceivable to have very long-wavelength electromagnetic fields generated via spinodal instabilities from the dynamics of  $\phi$  as a possible source of seed magnetic fields for the galactic dynamo.

As we know, the issue of the origin of the observed cluster and galactic magnetic fields of about a few  $\mu$ G [10] remains a puzzle [11]. These magnetic fields could have been resulted from the amplification of a seed field of  $B_{seed} \sim 10^{-23} \text{G}$  on a comoving scale larger than Mpc via the so-called galactic dynamo effect. A number of scenarios have been proposed for generating seed fields in the early universe, mainly relying on non-equilibrium conditions such as inflation, the electroweak and the QCD phase transitions. After the phase transitions, the universe became a highly conducting plasma so that the magnetic flux which existed is frozen in, and the ratio of the magnetic energy density and the thermal background,  $\rho_B/\rho_{\gamma}$ , remains constant thereafter. The required  $B_{seed}$ translates into  $\rho_B \simeq 10^{-34} \rho_{\gamma}$ . However, it turns out that the generated fields in these models are too small, except in somewhat contrived cases, to be of cosmological interest. In an equilibrium condition, a large damping term induced by the high plasma conductivity suppresses significantly any electromagnetic field fluctuations [12,13].

In this *Letter*, we will investigate the implication of the electromagnetic coupling of the evolving  $\phi$  field to the origin of the primordial magnetic field (PMF). Then,

we will provide a mechanism in which the PMF can be generated via the  $\phi$ -photon resonant conversion during the LSS formation.

Here we consider the  $\phi$ -photon coupling,

$$L_{\phi\gamma} = \frac{c}{f} \phi \ \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}, \tag{1}$$

where  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ , f is a mass scale, and c is a coupling constant which we treat as a free parameter. For the present consideration, we pick f equal to the reduced Planck mass  $M_p \equiv (8\pi G)^{-1/2}$ . We are thus led to study the cosmological evolution of the  $\phi$ -photon system in a flat universe. The effective action is

$$S = S_M + \int d^4x \sqrt{g} \left[ -\frac{R}{16\pi G} - \frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) + \frac{1}{\sqrt{g}} L_{\phi\gamma} \right], \tag{2}$$

where the signature is (-+++) and  $S_M$  is the classical action for matter. We assume that the universe today has matter  $\Omega_M^0 = 0.3$  and quintessence  $\Omega_\phi^0 = 0.7$ , and define an  $\Omega_\phi$ -weighted average [1]

$$\langle w_{\phi} \rangle = \int_{\eta_{ls}}^{\eta_{0}} \Omega_{\phi}(\eta) w_{\phi}(\eta) d\eta \times \left( \int_{\eta_{ls}}^{\eta_{0}} \Omega_{\phi}(\eta) d\eta \right)^{-1}, \quad (3)$$

where  $\eta_0$  and  $\eta_{ls}$  are respectively the conformal time today and at the last scattering, defined by  $\eta = H_0 \int dt a^{-1}(t)$  with the scale factor a and the Hubble constant  $H_0$ . Assuming a spatially homogeneous  $\phi$  field, the equation of motion is given by

$$\frac{d^2\theta}{d\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{d\theta}{d\eta} + \frac{a^2}{H_0^2 M_p^2}\frac{\partial V(\theta)}{\partial \theta} - \frac{c}{a^2 H_0^2 M_p^2}\langle F\tilde{F}\rangle = 0,$$
(4)

where  $\theta = \phi/M_p$ . The last term of Eq. (4) is the back reaction from the produced magnetic fields. Later we will show that this term is too small to have any effect on the evolution of the  $\phi$  field when we consider the photon production. For now we omit this term and thus the evolution of the cosmic background is governed by

$$\frac{d\bar{\rho}_{\phi}}{d\eta} = -3ah \left(1 + w_{\phi}\right) \bar{\rho}_{\phi},\tag{5}$$

$$\frac{dh}{d\eta} = -\frac{3}{2}ah^2 - \frac{1}{2}aw_{\phi}\bar{\rho}_{\phi},\tag{6}$$

where  $h = H/H_0$ ,  $\bar{\rho}_{\phi} = \rho_{\phi}/(M_p H_0)^2$ , and we have used  $\dot{\phi}^2 = (1 + w_{\phi})\rho_{\phi}$  and  $V(\phi) = (1 - w_{\phi})\rho_{\phi}/2$ . We have numerically solved the background equations by proposing a simple square-wave form for  $w_{\phi}$  as shown in Fig. 1. In order to satisfy the above-mentioned observational constraints on  $w_{\phi}$ , we have chosen  $w_{\phi} = -1$  for z < 2, and a width of the square-wave such that  $\langle w_{\phi} \rangle \simeq -0.7$ . Note

that at the present time  $\Omega_{\phi} = \bar{\rho}_{\phi}/(3h^2) = 0.7$  and  $H_0 t = 0.95$ . We have also plotted  $d\theta/d\eta = a\sqrt{(1+w_{\phi})\bar{\rho}_{\phi}}$ .

In the original PNGB models for dark energy [16], the well-known PNGB cosine potential was used. This results in that  $w_{\phi} > 0$  at the present time, which is disfavored by the observations. In fact, the square-wave equation of state is anticipated and quite general in the class of Q-models using PNGB fields incorporated with quantum corrections to the cosine potential [17]. In the PNGB models, the nonperturbatively large quantum fluctuations driven by spinoidal instabilities strongly dissipate the oscillation of the scalar field expectation value, leading to the field oscillation to become overdamped, and finally the field expectation value relaxes to the minimum of the effective action. However, we would like to emphasize here that the spinodal instabilities that we will discuss later to drive the amplitude fluctuations of long-wavelength magnetic fields to grow is a general phenomenon which is not restricted to such a square-wave equation of state. In fact, in the present consideration, the spinoidal instabilities responsible for the generation of the PMF occur during the formation of LSS for  $z \sim 10-2$  as long as during which the equation of state of the scalar field deviates from  $w_{\phi} = -1$ . After  $z \sim 2$ , the quintessential potential dominates and behaves like a cosmological constant with  $w \sim -1$  that ceases the photon production. Several quintessence models have been proposed to generate the equation of state of the scalar field that has the above-mentioned features. In Ref. [14], a potential of the form  $V(\phi) = V_n(\phi)e^{-\lambda\phi}$ , where  $V_p(\phi)$  is a polynomial of  $\phi$  and  $\lambda$  is a parameter, has been invented to introduce a local minimum in the exponential such that the field gets trapped into it. After the field gets trapped it starts behaving like a cosmological constant and the universe eventually enters an era of accelerated expansion. With suitable parameters, they showed that the equation of state stays near the value of about 0.3 for high redshifts and abruptly decreases to -1 for  $z \lesssim 3$ . This desired accelerated expansion can be also provided by a different  $V_p(\phi)$  arising from the interaction between branes in extra-dimension models [14], or having an oscillating term in the tracking oscillating model by Dodelson et al. in Ref. [2]. In k-essence models with a modified kinetic term for the scalar field, the equation of state suddenly drops to the value of about -1 at  $z \sim 2$  [15]. In summary, we have reviewed some quintessence models which have the desired evolution of the scalar field. With or without further tuning of the model parameters, they can be used to provide a cosmological background for the generation of the PMF during the LSS formation with the photon production ending at  $z \sim 3$ . However, for our purpose to illustrate how the PMF can be generated from the quintessence dynamics, we will use this simple, generic model in Fig. 1 that satisfies all existing observational constraints to carry out the calculation.

The most stringent limit on the  $\phi$ -photon coupling

comes from the cooling via the Primakoff conversion of horizontal branch (HB) stars in globular clusters [18],  $c/f < 1.5 \times 10^{-11} \text{GeV}^{-1}$ , which gives  $c < 3.7 \times 10^{7}$ . If  $\phi$ carries a mass  $m_{\phi} \sim H_0$ , it would decay into two photons with a width,  $\Gamma = c^2 m_{\phi}^3 / (4\pi f^2)$ . Hence the lifetime of  $\phi$  is much longer than the age of the universe. There is a very nice limit on c coming from the rotation of the plane of polarization of light from distant radio sources,  $c < 3 \times 10^{-2}/|\Delta\theta|$ , where  $\Delta\theta$  is the change in  $\theta$  between z=0.425 and today [9]. Since we have taken  $w_{\phi}=-1$ (i.e.,  $\Delta\theta = 0$ ) for z < 2, this limit does not apply to our case at all. From the theoretical point of view, it has been argued that because of radiative corrections cmust be infinitesimally small to keep the  $\phi$  field light [19]. However, in this study we would like to find out the value of the  $\phi$ -photon coupling needed in order to produce the large PMF over large correlation length scales if we assume that the generation of the PMF is solely due to the quintessence dynamics during the LSS formation.

From the action (2), the comoving magnetic field in the comoving coordinates  $(\tau, \mathbf{x})$  with  $\tau = \eta/H_0$  satisfies

$$\left(\nabla^{2} - \frac{\partial^{2}}{\partial \tau^{2}}\right) \mathbf{B} = \sigma a \left[\frac{\partial \mathbf{B}}{\partial \tau} - \nabla \times (\mathbf{v} \times \mathbf{B})\right] + 4c \frac{d\theta}{d\tau} \nabla \times \mathbf{B},$$
 (7)

where  $\nabla \equiv \partial/\partial \mathbf{x}$ ,  $\sigma$  is the plasma conductivity, and  $\mathbf{v} = d\mathbf{x}/d\tau$  is the peculiar plasma velocity. After hydrogen recombination, the residual ionization keeps the conductivity high,  $\sigma/H \sim 10^{22} (T/\text{eV})^{-3/2} \gg 1$ . As a result, the  $\sigma$  term under the assumption that  $\mathbf{v} = 0$  damps out any growth of the  $\mathbf{B}$  field on scales above  $\sim$  A.U. [12]. But this assumption may not hold when the universe has entered the non-linear regime. It could be understood from the recent study in Ref. [20] where the battery mechanism has been proposed as a source for generating a small initial magnetic field during LSS formation.

From the theory of small fluctuations,  $\mathbf{v}$  will grow with density perturbations. Velocity flows grow rapidly after extreme nonlinearities develop in the cosmic fluid. Therefore, the authors in Ref. [20] performed numerical simulations by adding the battery equation to the hydrodynamic code for structure formation, and showed that magnetic fields are built up in regions about to collapse into galaxies. The non-linear turbulence effect plays an important role in generating and subsequently amplifying the created PMF which overcomes the damping mechanism due to the high plasma conductivity. The numerical simulations showed that the rms value for  $\nabla \cdot \mathbf{v} \sim \nabla \times \mathbf{v}$  is about  $10^{-16}s^{-1}$  on comoving scale of Mpc at  $z \sim 3$  [20]. This is just the inverse of the time scale  $\tau_{LSS}$  for LSS formation. When a Mpc-scale magnetic field with magnitude B is simultaneously created, and as long as the growth rate  $\partial B/\partial \tau \sim B/\tau_{LSS}$  is comparable to the twisting term  $|\nabla \times (\mathbf{v} \times \mathbf{B})| \sim B \nabla \cdot \mathbf{v}$ , the  $\sigma$  term would be significantly reduced and the high plasma conductivity is no longer a hindrance to the growth of large-scale magnetic fields. Here we will show that the temporal variation of  $\phi$  can also be a generating source for large-scale magnetic fields. However, we will not pursue numerical simulations similar to Ref. [20], combining their battery source with the quintessence dynamics. As a first step, we will simply omit the  $\sigma$  term and solve for the photon equation self-consistently, while following closely their results and estimating the effect of the magneto-hydrodynamical dissipation. Of course, it would be very interesting to carry out direct numerical simulations by coupling the  $\phi$ -photon system in Eq. (7) to the hydrodynamic code to make a map of the created magnetic fields.

Now we write  $\mathbf{B} = \nabla \times \mathbf{A}_T$  and decompose the transverse field  $\mathbf{A}_T(\tau, \mathbf{x})$  into Fourier modes,

$$\mathbf{A}_{T} = \int \frac{d^{3}\mathbf{k}}{\sqrt{2(2\pi)^{3}k}} \left[ e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda=\pm} b_{\lambda\mathbf{k}} V_{\lambda\mathbf{k}}(\tau) \epsilon_{\lambda\mathbf{k}} + \text{h.c.} \right],$$
(8)

where  $b_{\pm \mathbf{k}}$  are destruction operators, and  $\epsilon_{\pm \mathbf{k}}$  are circular polarization unit vectors. Then, defining  $q = k/H_0$ , the mode equations are

$$\frac{d^2}{d\eta^2}V_{\pm q} + \left(q^2 \mp 4cq\frac{d\theta}{d\eta}\right)V_{\pm q} = 0,\tag{9}$$

with initial conditions at early epoch given by

$$V_{\pm q} = 1, \quad \frac{dV_{\pm q}}{d\eta} = -iq. \tag{10}$$

Hence, the comoving energy density of the magnetic field is given by  $\rho_B = \langle B^2 \rangle / 8\pi = \int dq (d\rho_B/dq)$  with

$$\frac{d\rho_B}{dq} = \frac{H_0^4}{32\pi^3} q^3 \coth\left[\frac{qH_0}{2T_0}\right] \sum_{\lambda=\pm} |V_{\lambda q}|^2, \tag{11}$$

where the coth term is the Bose-Einstein enhancement factor due to the presence of the CMB with current temperature  $T_0$  and energy density  $\rho_{\gamma} = \pi^2 T_0^4 / 15$ . The nonlinear growth in  $V_{+q}$  can be understood by treating the background field solution  $d\theta/d\eta > 0$  in Eq. (9) as a constant. For small fluctuations of the field, the longwavelength modes of  $V_{+q}$  , where q lies within the unstable band,  $q < 4c(d\theta/d\eta)$ , in fact "see" the inverted harmonic oscillators that cause the amplitude fluctuations to grow exponentially, while the evolution of  $V_{-a}$  modes is purely oscillating without any growth. The growth of these unstable modes is driven by spinodal instabilities [21]. Now we can write the approximate solution for the growing modes to Eq. (9) as  $V_{+q} \propto e^{\alpha\eta}$  where the exponent  $\alpha = \sqrt{4cq(d\theta/d\eta) - q^2}$  provided that q is in the unstable band. The maximum value of  $\alpha$  is  $\alpha_{max} =$  $2c(d\theta/d\eta)$  at  $q=2c(d\theta/d\eta)$ . Since the maximum growth for the amplitude fluctuations occurs at  $q = 2c(d\theta/d\eta)$ , in order to produce the large-scale correlated magnetic fields on 10 Mpc scales with the fastest growth, one can estimate the order-of-magnitude of the coupling c, which is about  $10^2$ , using the fact that  $d\theta/d\eta \simeq 0.8$  at redshift z=10 after which the universe was in the nonlinear regime and based on our previous argument that the high plasma conductivity effect which could damp out the produced magnetic fields can be consistently ignored. The self-consistency condition to reduce significantly this high conductivity effect can be justified in the sense that for the exponentially growing modes that we are interested in,  $\partial B/\partial \tau \sim \alpha_{\rm max} H_0 B \sim (10^{-16} s^{-1}) B$  with the coupling  $c \simeq 100$  and  $d\theta/d\eta \simeq 1$ , which is of the same order of the twisting term  $|\nabla \times (\mathbf{v} \times \mathbf{B})| \sim B \nabla \cdot \mathbf{v}$  during the LSS formation obtained from the numerical simulations [20].

As you will see in our numerical analysis, the spinodal instability provides a robust mechanism to generate the long-wavelength fluctuations where the created magnetic fields correspond to coherent collective behavior in which the fields correlate themselves over a large distance of order of 10 Mpc. However, since  $d\theta/d\eta$  is actually time dependent, the modes will grow only for a period of time before they move out of the unstable band. In the end, the growth of the long-wavelength modes will be shut off completely when the bandwidth of the unstable band vanishes as  $d\theta/d\eta$  reaches zero at redshift  $z \sim 4$ . We have numerically solved the mode equations (9) using c = 130and the background solution as shown in Fig. 1, and plotted the ratio  $(d\rho_B/dq)/\rho_{\gamma}$  in Fig. 2. Although photons are being produced as the scalar field starts rolling at  $z \sim 60$ , we have counted the photons produced only after z = 10 when the universe has presumably entered the non-linear regime. The result shows that a sufficiently large seed magnetic field of 10 Mpc scale has been produced before  $z \sim 4$ . Moreover, we notice that when  $c \sim 130$  the spinoidal instability gives rise to magnetic seed fields of the right magnitude and length scale. If we decrease the value of c, the peaks of the curves in Fig. 2 will shift to the lower left-hand corner.

The magnetic energy density can be approximately obtained from Fig. 2 and is about  $\Omega_B \simeq 10^{-33} \ll \Omega_{\phi}$ . Therefore, we anticipate that the back reaction effect from the produced magnetic fields on the scalar field evolution equation is negligible. This can be shown by evaluating the last term of Eq. (4) with

$$\langle F\tilde{F}\rangle = \frac{H_0^4}{\pi^2} \int dq q^2 \coth\left[\frac{qH_0}{2T_0}\right] \frac{d}{d\eta} \left(|V_{+q}|^2 - |V_{-q}|^2\right),\tag{12}$$

which is found to be extremely small compared to the other terms in Eq. (4) when the photons are being profusely produced.

Although the value of c=130 is well below the HB limit, undoubtedly it is much larger than the theoretical expectation. However, as suggested in Ref. [9], an unsuppressed  $\phi$ -photon coupling may arise in higher dimensional theories. One possible way to reduce the value

of the coupling and at the same time to produce the sufficiently large PMF on large correlation length scales during the LSS formation is to combine the mechanism we proposed here with the battery source in Ref. [20]. Also, note that the growth rate is controlled by the exponent  $\alpha \sim 2c(d\theta/d\eta)$ . As such, it may be possible to reduce the value of c by having a large  $d\theta/d\eta$ . In the case of scalar quintessence, we have seen in Fig. 1 that  $d\theta/d\eta < 0.8$  for z < 10. We cannot further increase  $d\theta/d\eta$  since  $w_{\phi}$  has already reached the maximum value. Perhaps, in the k-essence models, in which the kinetic term is modified to  $K(\phi)\dot{\phi}^2/2$  and we have  $\dot{\phi}^2 = (1+w_{\phi})\rho_{\phi}/K(\phi)$ , we may make  $d\theta/d\eta$  much larger than one by tuning the prefactor  $K(\phi)$ . This possibility is under investigation.

In conclusion, we have made an effort to link the darkenergy problem to a solution to another important problem in cosmology, namely, the generation of primordial magnetic fields. So far, we can only probe the equation of state of the dark energy through its cosmological effects. Undoubtedly, it is extremely important to have any clue about the nature of the dark energy. Here we have studied the possibility of generating the PMF on Mpc scales during the LSS formation by coupling the electromagnetic field to the evolving scalar field that accounts for the dark energy dynamics. The high conductivity effect due to residual ionization after hydrogen recombination can be argued to be significantly reduced as a result of the existence of the cosmic flow with nonlinear, twisting peculiar velocity to avoid a hindrance to the growth of the magnetic fields. We have shown that the nonlinear instability that drives the rapid growth of magnetic fields is of spinodal instability where the long-wavelength modes about the Mpc scales evolve as being the inverted harmonic oscillators and the amplitude fluctuations begin to grow up to a time at which the scalar field velocity approaches zero at redshift  $z \sim 4$ . We found that when  $c \sim 130$  the spinoidal instability gives rise to magnetic seed fields of the right magnitude and length scale. This strong coupling is well below the HB limit, but much larger than the theoretical expectation.

Here we have used a standard scalar quintessence and a simple equation of state as a working model. It is interesting to extend the idea to non-standard scalar models such as k-essence to see if the required  $\phi$ -photon coupling can be reduced to a smaller value with the specific equation of state. On the other hand, new laboratory searches for the coupling of photon to pseudo-scalar and more ingenious astrophysical or cosmological limits on the coupling would be needed. In addition, future CMB and SNe experiments would put a strong constraint on the quintessential equation of state at high-redshifts. In particular, combining the battery mechanism in Ref. [20] with the  $\phi$ -photon coupling which provides an alternative robust mechanism to generate the PMF in the hydrodynamical simulations would be a very interesting subject to tackle.

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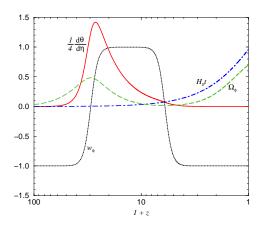


FIG. 1. The quantities  $d\theta/d\eta$ ,  $w_{\phi}$ ,  $H_0t$ , and  $\Omega_{\phi}$  as a function of redshift. Note that  $d\theta/d\eta$  is drawn 4 times smaller.

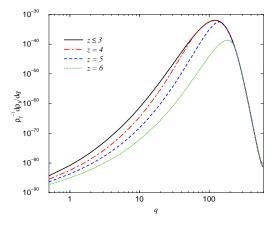


FIG. 2. Ratios of the spectral magnetic energy density to the present CMB energy density at various redshifts. The present wavelength of the magnetic field is given by  $2\pi/(qH_0)$ .

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